1.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \ldots$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

a) $f(x) = \dfrac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)}$

i) $\dfrac{50x^2+38x+9}{(5x+2)^2(1-2x)} = \dfrac{A}{5x+2} + \dfrac{B}{(5x+2)^2} + \dfrac{C}{1-2x}$

$50x^2 + 38x + 9 = A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$

multiplied by $(5x+2)^2(1-2x)$ ①

sub in $x = \frac{1}{2}$:

$50\left(\frac{1}{2}\right)^2 + 38\left(\frac{1}{2}\right) + 9 = C\left(5\left(\frac{1}{2}\right)+2\right)^2$

$\frac{81}{2} = \frac{81}{4}C$

$C = 2$

$50x^2 + 38x + 9 = A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$

sub in $x = -\frac{2}{5}$:

$50\left(-\frac{2}{5}\right)^2 + 38\left(-\frac{2}{5}\right) + 9 = B\left(1 - 2\left(-\frac{2}{5}\right)\right)$

$\frac{9}{5} = \frac{9}{5}B$

$B = 1$ ①

ii) sub in $x = 0$: $9 = 2A + B + 4C$ ①

$9 = 2A + 1 + 8$

$2A = 0$

$A = 0$ ①

$f(x) = \frac{1}{(5x+2)^2} + \frac{2}{1-2x}$

bi) $f(x) = (5x+2)^{-2} + 2(1-2x)^{-1}$

$(5x+2)^{-2} = \left(2\left[\frac{5x}{2} + 1\right]\right)^{-2} = 2^{-2}\left(1 + \frac{5}{2}x\right)^{-2}$ ①

$\left(1 + \frac{5}{2}x\right)^{-2} = 1 - 2\left(\frac{5}{2}x\right) + \frac{(-2)(-3)}{2!}\left(\frac{5}{2}x\right)^2 + \dots$ ①

$= 1 - 5x + \frac{75}{4}x^2 + \dots$

$\therefore 2^{-2}\left(1 + \frac{5}{2}x\right)^{-2} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots$ ①

$$(1-2x)^{-1} = 1 + 2x + \frac{(-1)(-2)}{2!}(-2x)^2 + \dots$$ 1

$$= 1 + 2x + 4x^2 + \dots$$

$$\therefore\ 2(1-2x)^{-1} = 2 + 4x + 8x^2 + \dots$$

$$f(x) = \frac{1}{(5x+2)^2} + \frac{2}{1-2x}$$

$$= \left[\frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots\right] + \left[2 + 4x + 8x^2 + \dots\right]$$ 1

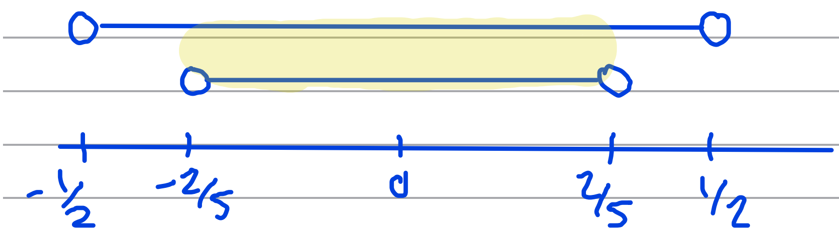
$$= \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + \dots$$ 1

b ii) $\left(1+\frac{5}{2}x\right)^{-2}$ is valid for $\left|\frac{5x}{2}\right| < 1$ $\times \frac{2}{5}$

$|x| < \frac{2}{5}$

$(1-2x)^{-1}$ is valid for $|-2x| < 1$

$|x| < \frac{1}{2}$



want the overlap region, so

$|x| < \frac{2}{5}$ 1

2. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

a) $\sqrt{4-9x} \equiv (4-9x)^{\frac{1}{2}}$

$\equiv \left\{4\left(1-\frac{9x}{4}\right)\right\}^{\frac{1}{2}}$

$\equiv 4^{\frac{1}{2}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$

$\equiv 2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$ 1

$= 2\left\{1 + \frac{\left(\frac{1}{2}\right)\left(-\frac{9}{4}x\right)}{1!} + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(-\frac{9}{4}x\right)^2}{2!} + \right.$ 1

$\left. \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(-\frac{9}{4}x\right)^3}{3!} + \ldots\right\}$ 1

$= 2\left\{1 - \frac{9x}{8} - \frac{81x^2}{128} - \frac{729x^3}{1024} + \ldots\right\}$

$= 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512} + \ldots$ 1

valid for $|x| < \frac{4}{9}$

b) overestimate. The terms that have been omitted will all be negative. (1)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

(4)

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2} \, dx$$

giving your answer to 4 significant figures.

(4)

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2} \, dx$$

giving your answer in the form $a \ln b + c$, where a, b and c are constants to be found.

(5)

a) $(3+x)^{-2} = \left(3\left(1+\frac{x}{3}\right)\right)^{-2} = 3^{-2}\left(1+\frac{x}{3}\right)^{-2}$

$= \frac{1}{9}\left(1 - \frac{2x}{3} + \frac{(-2)(-2-1)}{2!}\left(\frac{x}{3}\right)^2 + \ldots\right)$ (1) (1) (1)

$= \frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}$ (1)

b) $\int 6x(3+x)^{-2}\,dx \approx \int 6x\left(\frac{1}{9} - \frac{2x}{27} - \frac{x^2}{27}\right) dx$

$= \int\left(\frac{2x}{3} - \frac{4x^2}{9} + \frac{2x^3}{9}\right) dx$ (1)

$$\int_{0.2}^{0.4}\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx=\left[\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}\right]_{0.2}^{0.4}\quad 1$$

$$=\left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right)-\left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right)\quad 1$$

$$=\frac{223}{6570}$$

$$=0.03304\ (4sf)\quad 1$$

c) $\displaystyle\int_{0.2}^{0.4}\frac{6x}{(3+x)^2}\,dx$ let $u=3+x \Rightarrow x=u-3$

$du=dx$

limits: $(0.2,0.4)\rightarrow(3.2,3.4)$

$$=\int_{3.2}^{3.4}\frac{6(u-3)}{u^2}\,du=\int_{3.2}^{3.4}\left(\frac{6}{u}-18u^{-2}\right)du\quad 1$$

$$=\left[6\ln|u|+18u^{-1}\right]_{3.2}^{3.4}\quad 1\quad 1$$

$$=\left(6\ln 3.4+\frac{18}{3.4}\right)-\left(6\ln 3.2+\frac{18}{3.2}\right)\quad 1\quad =6\ln\left(\frac{17}{16}\right)-\frac{45}{136}\quad 1$$